5.7 A2 MODULE 4727: FURTHER PURE MATHEMATICS 3 (FP3)

Preamble

Knowledge of the specification content of Modules *C1, C2, C3, C4* and *FP1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *FP3*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Differential Equations

An integrating factor for $\frac{dy}{dx} + P(x)y = Q(x)$ is $e^{\int P(x)\,dx}$

Vectors

The plane through **a** with normal vector **n** is $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$

Complex Numbers

If $z = e^{i\theta}$ then: $\cos\theta = \frac{1}{2}\left(z + \frac{1}{z}\right)$

$$\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$$

Differential Equations

Candidates should be able to:

(a) find an integrating factor for a first-order linear differential equation, and use an integrating factor to find the general solution;

(b) use a given substitution to reduce a first-order differential equation to linear form or to a form in which the variables are separable;

(c) recall the meaning of the terms ‘complementary function’ and ‘particular integral’ in the context of linear differential equations, and use the fact that the general solution is the sum of the complementary function and a particular integral;

(d) find the complementary function for a first or second order linear differential equation with constant coefficients;

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(e) recall the form of, and find, a particular integral for a first or second order linear differential equation in the cases where $ax+b$ or $a\mathrm{e}^{bx}$ or $a\cos px + b\sin px$ is a suitable form, and in other cases find the appropriate coefficient(s) given a suitable form of particular integral;

(f) use initial conditions to find a particular solution to a differential equation, and interpret the solution in the context of a problem modelled by the differential equation.

Vectors

Candidates should be able to:

(a) understand the significance of all the symbols used when the equation of a line is expressed in the form $\dfrac{x-a}{p}=\dfrac{y-b}{q}=\dfrac{z-c}{r}$;

(b) understand the significance of all the symbols used when the equation of a plane is expressed in any of the forms $ax+by+cz=d$ or $(\mathbf{r}-\mathbf{a}).\mathbf{n}=0$ or $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$;

(c) recall the definition, in geometrical terms, of the vector product of two vectors, and, in cases where **a** and **b** are expressed in component form, calculate $\mathbf{a}\times\mathbf{b}$ in component form;

(d) use equations of lines and planes to solve problems concerning distances, angles and intersections, and in particular

(i) determine whether a line lies in a plane, is parallel to a plane, or intersects a plane, and find the point of intersection of a line and a plane when it exists,

(ii) find the line of intersection of two non-parallel planes,

(iii) find the perpendicular distance from a point to a plane, and from a point to a line,

(iv) find the angle between a line and a plane, and the angle between two planes,

(v) find the shortest distance between two skew lines.

Complex Numbers

Candidates should be able to:

(a) carry out operations of multiplication and division of two complex numbers expressed in polar form $\left(r(\cos\theta+\mathrm{i}\sin\theta)\equiv r\,\mathrm{e}^{\mathrm{i}\theta}\right)$, and interpret these operations in geometrical terms;

(b) understand de Moivre’s theorem, for positive and negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers;

(c) use de Moivre’s theorem to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle;

(d) use expressions for $\sin\theta$ and $\cos\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$, e.g. in expressing powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles or in summing series;

(e) find and use the *n*th roots of unity, e.g. to solve an equation of the form $z^n = a+\mathrm{i}b$.

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Groups

Candidates should be able to:

(a) recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set, and for which every element has an inverse in the set;

(b) use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, groups of matrices, transformations, integers modulo n);

(c) use algebraic methods to establish properties in abstract groups in easy cases, e.g. to show that any group in which every element is self-inverse is commutative;

(d) recall the meaning of the term 'order', as applied both to groups and to elements of a group, and determine the order of elements in a given group;

(e) understand the idea of a subgroup of a group, find subgroups in simple cases, and show that given subsets are, or are not, (proper) subgroups;

(f) recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (the proof of the theorem is not required);

(g) recall the meaning of the term 'cyclic' as applied to groups, and show familiarity with the structure of finite groups up to order 7 (questions on groups of higher order are not excluded, but no particular prior knowledge of such groups is expected);

(h) understand the idea of isomorphism between groups, and determine whether given finite groups are, or are not, isomorphic.