5.6 A2 MODULE 4726: FURTHER PURE MATHEMATICS 2 (FP2)

Preamble

Knowledge of the specification content of Modules *C1, C2, C3, C4* and *FP1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *FP2.*

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Hyperbolic Functions

$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$

$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$

$\tanh x \equiv \dfrac{\sinh x}{\cosh x}$

Rational Functions and Graphs

Candidates should be able to:

(a) express in partial fractions a rational function in which the denominator may include a factor of the form $(x^2 + a^2)$ in addition to linear factors as specified in section 5.4, and in which the degree of the numerator may exceed the degree of the denominator;

(b) determine the salient features of the graph of a rational function for which the numerator and denominator are of degree at most 2, including in particular

  (i) asymptotic behaviour (understanding of oblique asymptotes, as well as asymptotes parallel to the axes, is expected),

  (ii) any restrictions on the values taken by the function;

(c) understand and use the relationship between the graphs of $y = f(x)$ and $y^2 = f(x)$.

Polar Coordinates

Candidates should be able to:

(a) understand the relations between cartesian and polar coordinates (using the convention $r \geq 0$), and convert equations of curves from cartesian to polar form and vice versa;

(b) sketch simple polar curves, for $0 \leq \theta < 2\pi$ or $-\pi < \theta \leq \pi$ or a subset of either of these intervals, and identify significant features of polar curves such as symmetry, least/greatest values of *r*, and the form of the curve at the pole (knowledge that any values of θ for which $r = 0$ give directions of tangents at the pole is included);

(c) use the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, \mathrm{d}\theta$ for the area of a sector in simple cases.

Hyperbolic Functions

Candidates should be able to:

(a) recall definitions of the six hyperbolic functions in terms of exponentials, and sketch the graphs of simple hyperbolic functions;

(b) derive and use identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\sinh 2x \equiv 2\sinh x \cosh x$;

(c) use the notations $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$ to denote the principal values of the inverse hyperbolic relations, and derive and use expressions in terms of logarithms for these.

Differentiation and Integration

Candidates should be able to:

(a) derive and use the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$;

(b) derive and use the derivatives of $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$;

(c) use the first few terms of the Maclaurin series of e^x, $\sin x$, $\cos x$ and $\ln(1+x)$;

(d) derive and use the first few terms of the Maclaurin series of simple functions, e.g. $\sin x$, $\cos 3x$, $\mathrm{e}^x \sin x$, $\ln(3+2x)$ (derivation of a general term is not included);

(e) integrate $\dfrac{1}{\sqrt{a^2 - x^2}}$, $\dfrac{1}{a^2 + x^2}$, $\dfrac{1}{\sqrt{x^2 - a^2}}$ and $\dfrac{1}{\sqrt{x^2 + a^2}}$, and use appropriate trigonometric or hyperbolic substitutions for the evaluation of definite or indefinite integrals (the substitution $t = \tan \frac{1}{2} x$ is included);

(f) derive and use reduction formulae for the evaluation of definite integrals in simple cases;

(g) understand how the area under a curve may be approximated by areas of rectangles, and use rectangles to estimate or set bounds for the area under a curve or to derive inequalities concerning sums.

Numerical Methods

C3.3 IT3.2 IT3.3

Candidates should be able to:

(a) understand, in geometrical terms involving ‘staircase’ and ‘cobweb’ diagrams, the convergence (or not) of an iteration of the form $x_{n+1} = \mathrm{F}(x_n)$ to a root of the equation $x = \mathrm{F}(x)$;

(b) use the facts that, for an iteration $x_{n+1} = \mathrm{F}(x_n)$ which converges to α , successive (small) errors e_n are such that:

   (i) $\mathrm{e}_{n+1} \approx \mathrm{F}'(\alpha)\mathrm{e}_n$, if $\mathrm{F}'(\alpha) \neq 0$,

   (ii) e_{n+1} is approximately proportional to e_n^2 (in general) if $\mathrm{F}'(\alpha) = 0$;

(c) understand, in geometrical terms, the working of the Newton-Raphson method, and appreciate conditions under which the method may fail to converge to the desired root;

(d) derive and use iterations based on the Newton-Raphson method, and understand that this method is an example of an iteration of the form $x_{n+1} = \mathrm{F}(x_n)$ with $\mathrm{F}'(\alpha) = 0$.