

5.4 A2 MODULE 4724: CORE MATHEMATICS 4 (C4)

Preamble

Knowledge of the specification content of Modules *C1*, *C2* and *C3* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *C4*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Differentiation and Integration

$$\text{If } y = \sin kx \text{ then } \frac{dy}{dx} = k \cos kx$$

$$\text{If } y = \cos kx \text{ then } \frac{dy}{dx} = -k \sin kx$$

$$\int \cos kx \, dx = \frac{1}{k} \sin kx + c$$

$$\int \sin kx \, dx = -\frac{1}{k} \cos kx + c$$

$$\int f(g(x))g'(x) \, dx = f(g(x)) + c$$

Vectors

$$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$$

$$(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = ax + by + cz$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

Equation of a line through \mathbf{a} parallel to \mathbf{b} is $\mathbf{r} = \mathbf{a} + t\mathbf{b}$

Algebra and Graphs

Candidates should be able to:

- (a) simplify rational expressions, including factorising and cancelling;
- (b) divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero);
- (c) recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than
 - (i) $(ax + b)(cx + d)(ex + f)$,
 - (ii) $(ax + b)(cx + d)^2$,and where the degree of the numerator is less than that of the denominator;
- (d) use the expansion of $(1 + x)^n$ where n is a rational number and $|x| < 1$ (finding a general term is not included, but adapting the standard series to expand, e.g. $(2 - \frac{1}{2}x)^{-1}$ is included);
- (e) understand the use of a pair of parametric equations to define a curve, and use a given parametric representation of a curve in simple cases;
- (f) convert the equation of a curve between parametric and cartesian forms.

Differentiation and Integration

Candidates should be able to:

- (a) use the derivatives of $\sin x$, $\cos x$ and $\tan x$, together with sums, differences and constant multiples;
- (b) find and use the first derivative of a function which is defined parametrically or implicitly;
- (c) extend the idea of 'reverse differentiation' to include the integration of trigonometric functions (e.g. $\cos x$ and $\sec^2 2x$);
- (d) use trigonometric relations (such as double angle formulae) in order to facilitate the integration of functions such as $\cos^2 x$;
- (e) integrate rational functions by means of decomposition into partial fractions (restricted to the types of partial fractions specified above in *Algebra and graphs*);
- (f) recognise an integrand of the form $\frac{kf'(x)}{f(x)}$, and integrate, for example, $\frac{x}{x^2 + 1}$ or $\tan x$;
- (g) recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, for example, $x \sin 2x$, $x^2 e^x$, $\ln x$ (the relationship between integration by parts and differentiation of a product should be understood);
- (h) use a given substitution to simplify and evaluate either a definite or an indefinite integral (the relationship between integration by substitution and the chain rule should be understood).

First Order Differential Equations

Candidates should be able to:

- (a) formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality;
- (b) find by integration a general form of solution for a differential equation in which the variables are separable;
- (c) use an initial condition to find a particular solution of a differential equation;
- (d) interpret the solution of a differential equation in the context of a problem being modelled by the equation.

Vectors

Candidates should be able to:

- (a) use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, \overline{AB} , \mathbf{a} ;
- (b) carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms;
- (c) use unit vectors, position vectors and displacement vectors;
- (d) calculate the magnitude of a vector, and identify the magnitude of a displacement vector \overline{AB} as being the distance between the points A and B ;
- (e) calculate the scalar product of two vectors (in either two or three dimensions), and use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors;
- (f) understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$;
- (g) determine whether two lines are parallel, intersect or are skew;
- (h) find the angle between two lines, and the point of intersection of two lines when it exists.