Poisson - Hypothesis Testing

8 (i) On average I receive 8 letters every weekday. On a randomly chosen Monday I received 2 letters.

Jan 04

(a) Use a Poisson distribution to test, at a significance level as close to 5% as possible, whether this is evidence that I receive fewer than 8 letters on a Monday. State your hypotheses clearly. [5]

(b) Find the actual significance level of this test. [2]

4 At one time on motorways in the UK the average number of potholes was 6 per mile. One randomly chosen mile of the M1 was found to have 9 potholes. Test, at the 5% significance level, whether this evidence suggests that the average number of potholes per mile of the M1 was greater than 6. State your hypotheses clearly, and show the relevant probabilities. [6]

Jan 02

2 The average number of weeds per square metre of a large lawn, measured each May over many years, is known to be 4.0. One May a randomly chosen square metre of lawn is found to contain 7 weeds. Using the Poisson tables, test, at the 5% significance level, whether this is evidence of an increase in the mean number of weeds per square metre of the lawn, stating your hypotheses clearly. [6]

June 03

7 In excavating an archaeological site, Roman coins are found scattered throughout the site.

June 05

(i) State two assumptions needed to model the number of coins found per square metre of the site by a Poisson distribution. [2]

Assume now that the number of coins found per square metre of the site can be modelled by a Poisson distribution with mean λ.

(ii) Given that $\lambda = 0.75$, calculate the probability that exactly 3 coins are found in a region of the site of area 7.20 m^2. [3]

A test is carried out, at the 5% significance level, of the null hypothesis $\lambda = 0.75$, against the alternative hypothesis $\lambda > 0.75$, in Region LVI which has area 4 m^2.

(iii) Determine the smallest number of coins that, if found in Region LVI, would lead to rejection of the null hypothesis, stating also the values of any relevant probabilities. [4]

(iv) Given that, in fact, $\lambda = 1.2$ in Region LVI, find the probability that the test results in a Type II error. [3]

4 In a blood test, some blood is placed on a microscope slide and the number of corpuscles in each grid square of the slide is counted. The number of corpuscles per grid square in a sample of blood is a random variable with the distribution Po(μ).

Jun 04

(i) For healthy blood, it is known that $\mu = 2.0$. Find the probability that, in a randomly chosen sample of healthy blood, the number of corpuscles counted in one grid square is less than 3. [2]

(ii) A significance test of the null hypothesis $H_0 : \mu = 2.0$ as opposed to the alternative hypothesis $H_1 : \mu < 2.0$ is carried out, using a significance level as close as possible to 5%. The test is based on the total number of corpuscles counted in a group of 4 grid squares.

(a) Find the largest total number of corpuscles counted that would result in rejection of the null hypothesis. You should show the value of any relevant probability. [2]

(b) Given that, in fact, $\mu = 1.75$, find the probability that the test results in a Type II error. [3]