Normal - Hypothesis Testing

7 The mean solubility rating of widgets inserted into beer cans is thought to be 84.0, in appropriate units. A random sample of 50 widgets is taken. The solubility ratings, x, are summarised by

Jun 02

$$n = 50, \qquad \Sigma x = 4070, \qquad \Sigma x^2 = 336\,100.$$

Test, at the 5% significance level, whether the mean solubility rating is less than 84.0. [10]

8 A psychologist is testing the effect of background music on students' work. She knows that the average time for a randomly chosen student to complete a particular task in the absence of background music is 38.5 seconds. A sample of 50 students took the test with background music being played. The times taken, t seconds, are summarised as follows.

June 04

$$n = 50, \ \Sigma t = 1967, \ \Sigma t^2 = 77\,959.$$

✓ (i) Test, at the 5% significance level, whether the presence of background music has an effect on the times taken by students to complete the task. State your hypotheses clearly. [10]

✓ (ii) Give a reason why it is necessary to use the Central Limit Theorem in carrying out your test. [1]

6 A company manufactures electronic resistors. The resistances of the resistors are normally distributed with mean μ ohms and standard deviation 2.4 ohms. A random sample of 16 resistors is used to test the null hypothesis $\mu = 100$ against the alternative hypothesis $\mu \neq 100$ using a 5% significance level. The mean of the sample is denoted by $\bar{x}$.

Jun 02

(i) Determine, correct to 2 decimal places, the range of values of $\bar{x}$ for which the null hypothesis would be rejected. [4]

(ii) Calculate the probability of making a Type II error if the value of μ is in fact 102. [3]

(iii) The company uses four different machines to manufacture the resistors. Three are correctly adjusted, so that they produce resistors with mean 100 ohms, but one is incorrectly adjusted, so that it produces resistors with mean 102 ohms. A sample of 16 resistors is selected at random from the output of one of the machines, chosen at random. Using the rejection region (critical region) found in part (i), calculate the probability that the outcome of the test is to reject the hypothesis $\mu = 100$. [3]

5 Red lights used for traffic lights have a colour measured by wavelength L, in suitable units. It is known that the random variable L has the distribution $N(\mu, 10^2)$. For those lights in standard use, the value of μ is 660. A random sample of n lights is taken from the output of a new manufacturer. The sample mean wavelength is 657. A test is carried out, at the 5% significance level, of the hypothesis $H_0 : \mu = 660$ as opposed to $H_1 : \mu \neq 660$, for this manufacturer.

Jan 04

(i) The result of the test is to reject H_0. Calculate the smallest possible value of n. [5]

(ii) State the probability of a Type I error occurring as a result of the test. [1]

(iii) Given that a Type I error does occur, state what can be said about the null hypothesis. [1]

Normal - hyp testing (cont 1)

4 The height of sweet pea plants grown in a nursery is a random variable. A random sample of 50 plants is measured and is found to have a mean height 1.72 m and variance 0.0967 m^2.

June 05

(i) Calculate an unbiased estimate for the population variance of the heights of sweet pea plants. [2]

(ii) Hence test, at the 10% significance level, whether the mean height of sweet pea plants grown by the nursery is 1.8 m, stating your hypotheses clearly. [7]

5 A random variable X has mean μ. It is desired to test the null hypothesis $H_0 : \mu = 48$ as opposed to the alternative hypothesis $H_1 : \mu \neq 48$. A random sample of size 36 can be summarised by $\Sigma x = 1665$, $\Sigma x^2 = 77\,890$.

Jan 03

(i) Carry out the test, using a 2% significance level. [8]

(ii) State with a reason whether, in carrying out this test, it is necessary to assume that X has a normal distribution. [1]

5 A company manufactures CDs.

June 03

(a) The percentage X of sulphur-based compounds in the ink used for CD labels is modelled by the distribution $N(\mu, 0.4^2)$. If $\mu > 1.5$ the CDs will be liable to a defect known as 'bronzing'. A random sample of 10 CDs is analysed and it is found that the sample mean is 1.70. Carry out a test, at the 5% significance level, of the null hypothesis $H_0 : \mu = 1.5$ against the alternative hypothesis $H_1 : \mu > 1.5$, stating your conclusion. [5]

5 A company manufactures ropes for use by climbers. The company claims that the ropes have a mean breaking strength of 13 000 newtons. The standard deviation of the breaking strength is known to be 200 newtons. The breaking strengths of a sample of 50 ropes are measured, in order to test whether the company's claim is justified.

June 01

(i) State why, in this context, a one-tail test is more appropriate than a two-tail test. [1]

(ii) The sample is found to have a mean breaking strength of 12 950 newtons. Carry out a significance test, at the 5% level, to decide whether the company's claim is justified. [6]

(iii) Name a theorem used in carrying out your test, and state why it is needed. [2]

6 (i) Explain what is meant by a Type I error. [1]

Jan 05

(ii) The continuous random variable X has the distribution $N(\mu, \sigma^2)$. A test of the hypothesis $H_0 : \mu = 25$ is carried out at the 5% significance level, once a day for 300 days. Given that on each day the value of μ is 25, use a normal approximation to find the probability that a Type I error is made on at least 20 days. [6]

(iii) Explain whether, in answering part (ii), it is necessary to assume that the outcomes of the tests are independent. [1]

Normal - hyp testing (cont 2)

8 (i) A random variable X has the distribution $N(\mu, \sigma^2)$. The mean of a sample of 5 observations of X is denoted by $\bar{X}$. State the distribution of $\bar{X}$, giving the values of any parameters. [2]

Jan 05

(ii) A group of scientists is attempting to identify subatomic particles called *ocrons*. Ocrons have a mean path length of less than 42 cm. The path lengths of a random sample of five particles thought to be ocrons are measured, and the mean path length of the sample is found to be 36.6 cm. Path lengths are known to be normally distributed random variables with standard deviation 8 cm. Carry out a test, at the 10% significance level, of whether the population mean path length is less than 42 cm, stating your hypotheses clearly. [7]

(iii) A second group of scientists carries out a test that is identical, except that they use a 5% significance level. If the mean observed path length of the particles is consistent with a population mean of less than 42 cm, the scientists will claim that the particles are ocrons. State what the use of this smaller significance level suggests about the intentions of the scientists in deciding whether or not to claim that the observed particles are ocrons. [2]

7 The time, T minutes, taken for a randomly chosen employee to complete a certain task can be modelled by a normal distribution with mean μ and standard deviation 7.0. It is known that, for experienced employees, the value of μ is 50.0. The times taken for a random sample of n newly qualified employees to complete the task are found. These times will be used in a test, at the 5% significance level, of whether newly qualified employees take longer than experienced employees.

Jan 03

(i) State appropriate hypotheses for the test. [1]

(ii) Given that $n = 40$ and that the mean time taken by the sample of newly qualified employees is 52.0 minutes, carry out the test, stating your conclusions clearly. [5]

(iii) The critical region for the test is $\{\overline{T} > c\}$. The test is to be modified so that the probability that it results in a Type I error is 0.05.

(a) Show that c and n would have to satisfy the equation

$$c - 50.0 = \frac{11.515}{\sqrt{n}},$$

approximately. [3]

It is also required that the probability that the test results in a Type II error when $\mu = 52.0$ is 0.05.

(b) Find a second (approximate) equation that would have to be satisfied by c and n. [2]

(c) Find the value of c from the equations in parts (a) and (b). [1]

(d) Hence find a suitable value for n. [2]