Continuous Random Variables

3 The lifetime, in years, of an electrical appliance may be modelled by the random variable T with probability density function

June 05

$$f(t) = \begin{cases} \dfrac{k}{t^2} & 1 \leqslant t \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{4}{3}$. [2]

(ii) Find the value of the mean of T, giving your answer in the form $a \ln b$. [3]

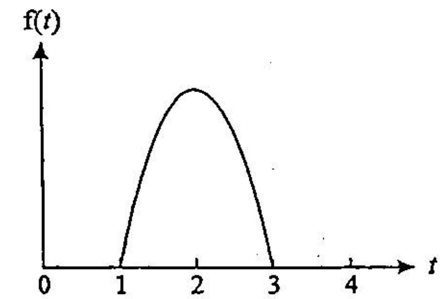
(iii) Find the time t_0 for which $P(T > t_0) = 0.1$. [3]

4 A student models the time, T hours, required for a certain journey by a continuous random variable with probability density function given by

Jan 03

$$f(t) = \begin{cases} k(t-1)(3-t) & 1 \leqslant t \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant. A sketch of this density function is shown in the diagram.



(i) Show that $k = \frac{3}{4}$. [2]

(ii) Write down the value of $E(T)$, and find the variance of T. [5]

(iii) Suggest one feature of the model which may not be realistic, and sketch the probability density function of a more realistic model. [2]

5 A continuous random variable X has the probability density function

Jan 02

$$f(x) = \begin{cases} \dfrac{a}{x^3} & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{9}{4}$. [3]

(ii) Find $E(X)$. [3]

(iii) Find the value w such that $P(X < w) = 0.3$. [3]

Continuous Random Variables (cont 1)

6 Archie is taking part in an archery competition. For each of his shots, the arrow strikes the target at a distance X metres from the centre, where the random variable X has probability density function given by

Jun 03

$$f(x) = \begin{cases} 12x^2(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

The score for each arrow depends on its distance from the centre of the target, as given in the following table, where the probabilities are given correct to 4 decimal places.

Distance x metres	$0 \leqslant x < 0.25$	$0.25 \leqslant x < 0.5$	$0.5 \leqslant x < 0.75$	$0.75 \leqslant x < 1$
Score s	40	30	20	10
$P(S = s)$	0.0508	a	b	0.2617

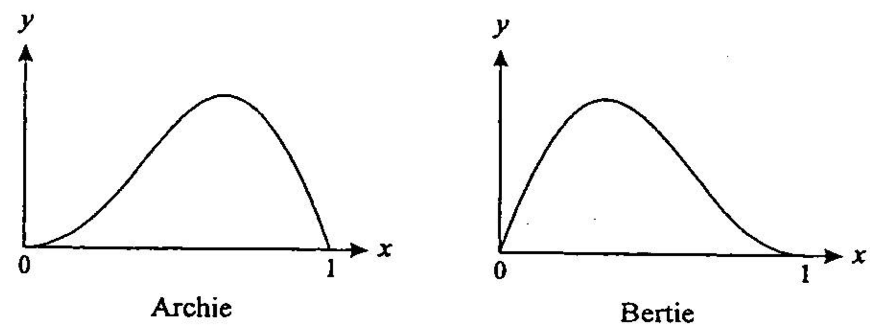
(i) Find the values of a and b, giving your answers correct to 4 decimal places. [5]

(ii) Find the expectation of

(a) X, [3]

(b) S. [2]

(iii) Bertie is also a competitor in the same competition. The graphs of the probability density functions for Archie and for Bertie are shown.



By consideration of the shapes of the graphs, state with a reason which of Archie and Bertie is likely to score more points in the competition. [2]

8 The time, in minutes, for which a customer is prepared to wait on a telephone complaints line is modelled by the random variable X. The probability density function of X is given by

Jun 02

$$f(x) = \begin{cases} \frac{4}{81}x(9-x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) (a) Show that the value y which satisfies $P(X < y) = \frac{3}{5}$ satisfies

$$5y^4 - 90y^2 + 243 = 0.$$ [3]

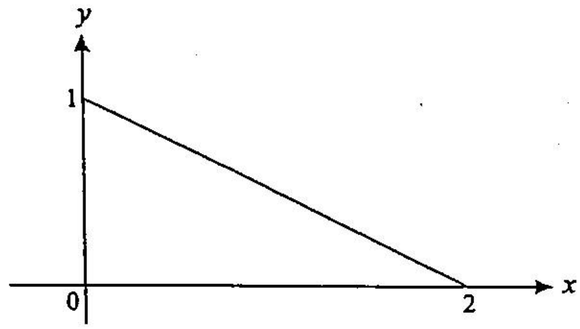
(b) Using the substitution $w = y^2$, or otherwise, solve the equation in part (a) to find the value of y. [3]

Continuous Random Variables (cont 2)

7 Two models are proposed for the continuous random variable X. Model 1 has probability density function

Jan 05

$$f_1(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$



The diagram shows the graph of $y = f_1(x)$.

(i) Find the upper quartile of X (i.e., find the value q such that $P(X < q) = 0.75$) according to model 1. [4]

Model 2 has probability density function

$$f_2(x) = \begin{cases} k(4 - x^2) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) The graph of $y = f_2(x)$ intersects the y-axis at the point $(0, 4k)$. Copy the diagram showing the graph of $y = f_1(x)$. On your copy sketch the graph of $y = f_2(x)$, explaining how you can tell without doing any integration that $4k < 1$. [4]

(iii) State whether the value of q obtained from model 1 is greater than, equal to, or less than the value given by model 2. Use your diagram to justify your answer. [2]

6 The continuous random variable X has probability density function given by

June 01

$$f(x) = \begin{cases} k(x-2)^2 & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) the value of k, [3]

(ii) $P(X < 2.5)$, [3]

(iii) $E(X)$, [4]

(iv) the value m such that $P(X < m) = 0.5$. [3]

Continuous Random Variables (cont 3)

7 Metal struts are designed to be of a given length, but in the manufacturing process the length is subject to an error represented by the continuous random variable X. Two models are proposed for X, with probability density functions as follows.

Jun 04

$$\text{Model 1:} \quad f_1(x) = \begin{cases} k_1 & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{Model 2:} \quad f_2(x) = \begin{cases} k_2(1-x^2) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k_1 and k_2 are constants.

(i) Sketch, on the same axes, the graphs of $f_1(x)$ and $f_2(x)$. [3]

(ii) For each of the two models, describe in everyday terms how the errors in the lengths vary. [2]

(iii) Write down the value of k_1. [1]

(iv) Calculate the standard deviation, σ, of Model 1. [4]

(v) Without further calculation, state with a reason whether the standard deviation of Model 2 is less than, equal to, or greater than σ. [2]

7 The random variable T represents the number of minutes that a train is late for a particular scheduled journey on a randomly chosen day.

June 04

(i) Give a reason why T could probably not be well modelled by a normal distribution. [1]

(ii) The following probability density function is proposed as a model for the distribution of T:

$$f(t) = \begin{cases} \frac{1}{67\,500}t(t-30)^2 & 0 \leqslant t \leqslant 30, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of $E(T)$. [3]

On a randomly chosen day I will allow for the train to be up to t_0 minutes late. I wish to choose the value of t_0 for which the probability that the train is less than t_0 minutes late is 0.95.

(b) Show that t_0 satisfies the equation

$$t_0^4 - 80t_0^3 + 1800t_0^2 = 256\,500. \quad [4]$$

(c) Show that the value of t_0 lies between 22 and 23. [2]