Binomial - Hypothesis Testing

4 A local government spokesman claims that at least three-quarters of the residents of a town are in favour of plans to build a new by-pass for the town. An opinion poll showed that 10 out of a random sample of 16 residents of the town were in favour of the plans. Test, at the 10% significance level, whether the results of the opinion poll are consistent with the spokesman's claim, stating your hypotheses clearly. [7]

Jan 05

7 At Celtic archaeological sites in a certain region, it is known that on average 30% of gold ornaments are from the Hallstadt culture and the remainder from other cultures. At a newly excavated site 240 gold ornaments were discovered, and of these 88 were from the Hallstadt culture. It may be assumed that these 240 ornaments are a random sample of all gold ornaments at the site. Use a suitable approximation to test, at the 1% significance level, whether this is evidence that more than 30% of all gold ornaments at the newly excavated site are from the Hallstadt culture. State your hypotheses clearly. [11]

Jan 02

4 An employee is accused by his employer of being late for work too often. The employee claims that, on average, he is late on no more than one day in ten. The employer finds that, over a random sample of 20 days, the employee is late on r days. The employer carries out a significance test, at the 5% level, to decide whether, on average, the employee is late on more than one day in ten.

June 01

(i) State suitable null and alternative hypotheses for the test. [2]

(ii) Find the set of values of r for which the null hypothesis would be rejected, and state the conclusion of the test in the case $r = 4$. [5]

(iii) Given that, in fact, the probability that the employee is late for work on a randomly chosen day is 0.2, find the probability of making a Type II error in the test. [2]

5 The proportion of left-handed adults in a country is known to be 15%. It is suggested that for mathematicians the proportion is greater than 15%. A random sample of 12 members of a university mathematics department is taken, and it is found to include five who are left-handed.

Jun 02

(i) Stating your hypotheses, test whether the suggestion is justified, using a significance level as close to 5% as possible. [7]

(ii) In fact the significance test cannot be carried out at a significance level of exactly 5%. State the probability of making a Type I error in the test. [2]

Binomial - hyp testing (cont 1)

7 A referendum is held, in which voters are required to answer Yes or No to a single question. The probability that a randomly chosen voter in the constituency of Coketown will answer Yes to the question is denoted by p.

June 03 (i) A random sample of 25 voters is chosen from Coketown. These voters are asked whether they intend to answer Yes or No.

(a) Find the critical region for a test, at the 5% significance level, of the null hypothesis $H_0 : p = 0.5$ against the alternative hypothesis $H_1 : p < 0.5$. You should state the values of any relevant probabilities obtained from tables. [3]

(b) State the conclusion of the test, in context, if 10 of the sample of 25 say they intend to answer Yes. [2]

It is known that, in two different constituencies, Abbotsea and Budmouth, the corresponding values of p are 0.4 and 0.5 respectively.

(ii) If the test described in part (i) is carried out in Abbotsea, find the probability that the result of the test is to accept the null hypothesis. [2]

One of the two constituencies, Abbotsea or Budmouth, is chosen at random (with equal probability), and from that constituency the test described in part (i) is carried out. Calculate the probability that

(iii) the result of the test is to accept the null hypothesis, [3]

(iv) the test leads to a Type II error. [3]

6 A factory makes chocolates of different types. The proportion of milk chocolates made on any day is denoted by p. It is desired to test the null hypothesis $H_0 : p = 0.8$ against the alternative hypothesis June 05 $H_1 : p < 0.8$. The test consists of choosing a random sample of 25 chocolates. H_0 is rejected if the number of milk chocolates is k or fewer. The test is carried out at a significance level as close to 5% as possible.

(i) Use tables to find the value of k, giving the values of any relevant probabilities. [3]

(ii) The test is carried out 20 times, and each time the value of p is 0.8. Each of the tests is independent of all the others. State the expected number of times that the test will result in rejection of the null hypothesis. [2]

(iii) The test is carried out once. If in fact the value of p is 0.6, find the probability of rejecting H_0. [2]

(iv) The test is carried out twice. Each time the value of p is equally likely to be 0.8 or 0.6. Find the probability that exactly one of the two tests results in rejection of the null hypothesis. [4]

5 (b) The probability that a randomly chosen CD manufactured by the company is defective is denoted by p. When the manufacturing process is working as intended, it is known that $p = 0.02$. Quality control is achieved by selecting a sample of 300 CDs and determining how many CDs in the Jun 03 sample are defective. A 5% significance test is then carried out of the null hypothesis $H_0 : p = 0.02$ against the alternative hypothesis $H_1 : p > 0.02$. Using a normal approximation, determine the critical region for the test. [6]