Approximations

2 In a large school, 5% of the pupils have gained a Silver Award. A random sample of 60 pupils is obtained, and the random variable R is the number of these pupils who have gained a Silver Award.

June 01

(i) State, with a reason, whether a Poisson approximation to the distribution of R would be appropriate. [3]

2 State with justification what approximation, if any, could be made to the distributions of the following random variables, stating the values of any parameters.

Jan 02

(i) A process which generates a random integer between 1 and 100, inclusive, is carried out 75 times. The random variable X is the number of integers generated that are less than or equal to 4. [3]

(ii) $X \sim \text{Po}(20)$. [2]

3 (i) State conditions under which the normal distribution can be used as an approximation to the distribution $\text{B}(n, p)$. [2]

Jan 03

A series of digital messages, each consisting of 8000 bits, is transmitted. For each bit, the probability that it is wrongly received is 0.005. A message is unreadable if more than 50 bits are wrongly received.

(ii) Using a suitable approximation, find the probability that a message is unreadable. [6]

a

1 Sixty people each make two throws with a fair six-sided die. Using a suitable approximation, calculate the probability that at least four of the sixty obtain two sixes. [5]

Jan 02

6 A hotel has 10 rooms, which are always occupied, and in each room there is a complimentary packet of biscuits. If on any night the packet is opened, it is replaced by a new packet next day. The hotel manager knows from experience that, for each room and for each night, the probability that the packet is opened is 0.4, independently of other rooms and other nights. He keeps spare packets in stock.

Jan 04

(i) The manager needs to be at least 95% sure that he has enough packets in stock each day to be able to replace those that are opened. Find the smallest number of packets that he needs to have in stock each day. [3]

(ii) At the start of a week (7 days), there are 35 packets in stock. Use a suitable approximation to calculate the probability that this is enough to meet the demands in the coming week. [6]

3 A fair six-sided die is thrown 540 times. Use a suitable approximation to calculate the probability that at least 80 sixes are obtained. [6]

June 01

1 The random variable X has a $\text{B}(60, 0.02)$ distribution. Use an appropriate approximation to find $\text{P}(X \leqslant 2)$. [3]

June 05

Approximations (cont 1)

3 On average 25% of pupils in a large school come to school by bus. Use a suitable approximation to find the probability that, in a random sample of 48 pupils, at least 15 come to school by bus. [6]

Jan 04

5 The random variable W has the distribution B(30, p).

June 05

(i) Use the exact binomial distribution to calculate P($W = 10$) when $p = 0.4$. [2]

(ii) Find the range of values of p for which you would expect that a normal distribution could be used as an approximation to the distribution of W. [3]

(iii) Use a normal approximation to calculate P($W \leqslant 10$) when $p = 0.4$. [5]

6 Requests for the services of a professional services company are received at a constant average rate of 38 per week, independently of one another. If more than 50 requests are received in any one week, the company has to take on extra staff.

Jan 04

(i) Use a suitable approximation to show that the probability that the company has to take on extra staff in one randomly chosen week is 0.021, correct to 3 decimal places. [5]

(ii) Use a suitable approximation to find the probability that, in a year of 50 working weeks, the company has to take on extra staff in more than 2 weeks. [5]

8 (ii) On Saturdays I also receive an average of 8 letters.

Jan 04

(a) State an assumption needed to model the total number of letters I receive on four consecutive Saturdays by a Poisson distribution. [1]

(b) Use a suitable approximation to the Poisson distribution to calculate the probability that, on four consecutive Saturdays, I receive a total of 25 or fewer letters. [5]

5 The random variable Y has a Poisson distribution with mean 20.

Jan 05

(i) Use the exact distribution to calculate P($Y = 22$). [2]

(ii) Use a suitable approximation to calculate P($Y = 22$). [4]

(iii) Calculate the percentage error in the approximate value. [2]

7 Over a long period it has been found that the number of typing errors made by a secretary is, on average, 6 per hour.

June 01

(i) State an assumption which you need to make in order to model the number of typing errors in a randomly chosen hour by a Poisson distribution. [1]

Assuming that this model is valid, and that the relevant periods of time are randomly chosen,

(ii) calculate the probability that, in one hour, the secretary makes more than 5 typing errors, [2]

(iii) calculate the probability that, in ten minutes, the secretary makes exactly two typing errors, [3]

(iv) use a suitable approximation to calculate the probability that, in four hours, the secretary makes more than 30 typing errors, [5]

(v) find the longest period of time, in minutes, for which the probability that the secretary makes no typing errors is greater than 0.9. [3]

6 On average a motorway police force records one car that has run out of petrol every two days.

Jun 02

(a) (i) Using a Poisson distribution, calculate the probability that, in one randomly chosen day, the police force records exactly two cars that have run out of petrol. [3]

(ii) Using a Poisson distribution and a suitable approximation to the binomial distribution, calculate the probability that, in one year of 365 days, there are fewer than 205 days on which the police force records no cars that have run out of petrol. [5]

(b) State an assumption needed for the Poisson distribution to be appropriate in part (a), and explain why this assumption is unlikely to be valid. [2]