Random Variables: Probability Distributions

Jun 01

4 (iii) The probability distribution of D is given in the table below.

d	0	1	2
$\mathrm{P}(D = d)$	$\frac{1}{16}$	$\frac{9}{16}$	$\frac{3}{8}$

Find $\mathrm{Var}(D)$. [4]

Spec

5 A sixth-form class consists of 7 girls and 5 boys. Three students from the class are chosen at random. The number of boys chosen is denoted by the random variable X. Show that

(i) $\mathrm{P}(X = 0) = \frac{7}{44}$, [2]

(ii) $\mathrm{P}(X = 2) = \frac{7}{22}$. [3]

The complete probability distribution of X is shown in the following table.

x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{7}{44}$	$\frac{21}{44}$	$\frac{7}{22}$	$\frac{1}{22}$

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [5]

Jun 03

5 A jar contains 3 red discs and 2 white discs. A disc is taken at random from the jar and its colour is noted. The disc is not replaced. This process is repeated until a white disc is taken. Let D be the number of discs taken, up to and including the white disc.

(i) Show that $\mathrm{P}(D = 2) = \frac{3}{10}$. [2]

(ii) Copy and complete the probability distribution table which is given in a partially completed form below. [3]

d	1	2	3	4
$\mathrm{P}(D = d)$	$\frac{2}{5}$	$\frac{3}{10}$		

(iii) Use the table found in part (ii) to calculate $\mathrm{E}(D)$ and $\mathrm{Var}(D)$. [5]

Jan 02

3 A discrete random variable X has the probability distribution given in the following table. It is given that $\mathrm{E}(X) = 0.95$.

x	-1	0	1	2	3
$\mathrm{P}(X = x)$	a	b	0.1	0.3	0.2

(i) Find the probability that X is greater than $\mathrm{E}(X)$. [1]

(ii) Find the values of a and b. [4]

(iii) Calculate $\mathrm{Var}(X)$. [3]

Probability Dists. (cont 1)

7 Two people, Ben and Chandra, were asked to rank 3 CDs, X, Y and Z, in order of preference. Ben stated that X was best, Y was second best and Z was the least good. Chandra chose her order of preference at random.

(i) How many different orders could Chandra choose? [1]

Let R be the value of Spearman's rank correlation coefficient between Chandra's order of preference and Ben's. The tables below show two of the possible orders Chandra could choose, and the resulting value of R.

Jan 03

Case 1

	X	Y	Z
Ben's ranking	1	2	3
Chandra's ranking	1	2	3

The resulting value of R is 1

Case 2

	X	Y	Z
Ben's ranking	1	2	3
Chandra's ranking	2	1	3

The resulting value of R is 0.5

(ii) Calculate the value of R for each of the other possible orders that Chandra could choose. [7]

(iii) Hence show that the distribution of R is given by the table below. [1]

r	-1	-0.5	0.5	1
$P(R = r)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

(iv) Find $E(R)$ and $Var(R)$. [5]

5 Christie throws 3 fair coins. She picks up any coins that show heads on this first throw and throws them a second time. Let X be the number of heads Christie obtains on her first throw. Let Y be the number of heads Christie obtains on her second throw. Then the total number of heads that Christie obtains altogether, H, is equal to $X + Y$.

Nov 04

(i) Show that $P(H = 1) = \frac{3}{16}$. [3]

The complete probability distribution table for H is shown below.

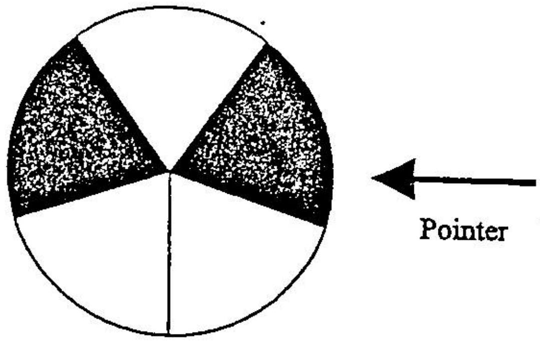
h	0	1	2	3	4	5	6
$P(H = h)$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{9}{32}$	$\frac{13}{64}$	$\frac{9}{64}$	$\frac{3}{64}$	$\frac{1}{64}$

(ii) Calculate $E(H)$. [2]

(iii) Calculate $Var(H)$. [3]

Probability Dists (cont 2)

3



Nov 03

The diagram shows a 'wheel of fortune' with 5 equal sectors. Three sectors are white and the other two are black. When the wheel is spun it eventually stops with the pointer pointing to one of the five sectors. Contestants spin the wheel until *either* they have obtained one result of each colour *or* until they have spun the wheel four times. Let S be the number of spins made by a randomly chosen contestant.

(i) Show that $P(S = 3) = \frac{6}{25}$. [3]

(ii) The probability distribution of S is given in the table below.

s	2	3	4
$P(S = s)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{7}{25}$

Find the variance of S. [4]

7 Jane and Dave each spin a coin 3 times. The coin which Jane uses is fair but Dave's coin is biased and for his coin the probability of turning up heads is $\frac{2}{3}$.

Let J be the number of heads that Jane obtains in 3 spins of her coin and let D be the number of heads that Dave obtains in 3 spins of his coin.

(i) Copy and complete the tables below to show the probability distributions of J and D. [5]

j	0	1	2	3
$P(J = j)$	$\frac{1}{8}$			

Jan 04

d	0	1	2	3
$P(D = d)$	$\frac{1}{27}$			

The random variable X is defined by the equation $X = J - D$.

(ii) Show that $P(X = 2) = \frac{1}{24}$. [4]

(iii) Calculate the probability that Jane and Dave obtain a total of at least 5 heads in their 6 spins. [4]

Probability Dists (cont 3)

May 02

2 A discrete random variable X has the probability distribution given by the table below. The expectation of this distribution is denoted by μ and the variance is denoted by σ^2.

x	2	3	4	5	6
$P(X = x)$	0.2	0.1	0.1	0.4	p

(i) Show that the value of p is 0.2. [2]

(ii) Find μ. [2]

(iii) Find σ^2. [3]

Jun 05

4 A 'turn' in a game begins by throwing two unbiased dice. If at least one of the dice shows a six then the turn ends. If neither dice shows a six then one of the dice is thrown again and the turn ends. The number of sixes obtained at the end of a turn is denoted by S. The following table gives the probability distribution of S.

s	0	1	2
$P(S = s)$	a	b	$\frac{1}{36}$

(i) Find the value of a and show that $b = \frac{85}{216}$. [3]

(ii) Find $E(S)$. [2]

(iii) Find the probability that, out of 5 turns, exactly 3 turns result in $S = 0$. [3]

Jan 01

5 A bag contains seven sweets identical in shape and size. Three of the sweets are lemon sweets and the other four are orange sweets. Ahmar keeps selecting sweets from the bag at random until he gets a lemon sweet. He does not return to the bag any orange sweets which he selects. The number of sweets selected up to and including the first lemon sweet is denoted by the random variable X.

(i) Show that

(a) $P(X = 2) = \frac{2}{7}$, [2]

(b) $P(X = 5) = \frac{1}{35}$. [2]

(ii) The distribution of X is shown in the following table.

x	1	2	3	4	5
$P(X = x)$	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{6}{35}$	$\frac{3}{35}$	$\frac{1}{35}$

Calculate

(a) $E(X)$, [2]

(b) $\text{Var}(X)$. [3]

Probability Dists (cont 4)

3 Paul plays a game in which a fair coin is spun 4 times. If the number of heads is 0 or 1, Paul loses £5, if the number of heads is 2, Paul wins £5 and if the number of heads is 3 or 4, Paul wins £10. Let £W be the amount which Paul wins in one randomly chosen game.

Jun 04

(i) Show that $P(W = -5) = \frac{5}{16}$. [2]

(ii) Copy and complete the table below to show the probability distribution of W. [2]

w	-5	5	10
$P(W = w)$	$\frac{5}{16}$		

(iii) Show that $E(W) = \frac{55}{16}$. [2]

(iv) Find $\mathrm{Var}(W)$. [3]