Geometric Distribution

2 The random variable X has a Geo(0.4) distribution.

Jun 03

(i) Show that $P(X = 4) = 0.0864$. [2]

(ii) Calculate the probability $P(4 \leqslant X < 9)$. [3]

(iii) State the value of $E(X)$. [1]

Spec 4 Each packet of the breakfast cereal Fizz contains one plastic toy animal. There are five different animals in the set, and the cereal manufacturers use equal numbers of each. Without opening a packet it is impossible to tell which animal it contains. A family has already collected four different animals at the start of a year and they now need to collect an elephant to complete their set. The family is interested in how many packets they will need to buy before they complete their set.

(i) Name an appropriate distribution with which to model this situation. State the value(s) of any parameter(s) of the distribution, and state also any assumption(s) needed for the distribution to be a valid model. [3]

(ii) Find the probability that the family will complete their set with the third packet they buy after the start of the year. [2]

(iii) Find the probability that, in order to complete their collection, the family will need to buy more than 4 packets after the start of the year. [3]

5 A jar contains 4 red discs and 5 white discs. A student plays a game in which each turn consists of selecting two discs at random from the jar, one after the other. The first disc is not replaced in the jar before the second is selected. The student wins a prize if both the discs selected are red. At the end of the turn both the discs are returned to the jar, ready for the next turn.

Nov 03

(i) Show that the probability that the student wins a prize on one turn is $\frac{1}{6}$. [2]

The student repeatedly has turns of the game until he wins a prize. Let X be the number of turns up to and including the first turn on which he wins a prize.

(ii) State the probability distribution of X, giving the value(s) of any parameter(s). [2]

(iii) Find $P(X = 4)$. [2]

(iv) State the value of $E(X)$. [1]

5 On a fairground stall the prize is a large cuddly toy worth £20. In order to win this prize Vicky needs to throw a hoop over a pole. For each throw, her probability of success is $\frac{2}{35}$, independently of all other throws. She pays the owner of the stall 50p for each throw and keeps throwing the hoop until she wins the prize.

May 02

(i) Calculate the probability that Vicky takes fewer than 3 throws to win the prize. [3]

(ii) Find the expected number of throws that Vicky needs to win the prize. [2]

(iii) Calculate the probability that Vicky pays more to win the prize than it is worth. [3]

Geometric (cont 1)

3 A man is standing at a bus stop waiting for a 'number 37' bus. He makes the following modelling assumptions:

- at any time the probability that the next bus to arrive will be a number 37 bus is $\frac{1}{12}$,
- whether the next bus to arrive is a number 37 bus or not is independent of all other arrivals.

Jan 04

Starting from the time when the man first arrives at the bus stop, let X be the number of buses which arrive up to and including the first number 37 bus.

(i) State the distribution of X, giving the value(s) of any parameter(s). [2]

(ii) Calculate

(a) $\mathrm{P}(X = 3)$, [2]

(b) $\mathrm{P}(X > 5)$. [2]

(iii) State the value of $\mathrm{E}(X)$. [1]

(iv) Give one reason why the modelling assumptions may not be correct. [1]

Jun 01

1 Every time a player throws a dart at a dartboard, the probability that she scores a 'double' is $\frac{1}{8}$ independently of the result of any other throw. She keeps throwing until she scores a double. Let T be the number of throws taken by the player up to and including the throw with which she first scores a double.

(i) Name the distribution of T, and find $\mathrm{E}(T)$. [3]

(ii) Find $\mathrm{P}(T = 3)$. [3]

Nov 02

6 (a) A jar contains 3 red balls and 2 white balls. A ball is selected at random from the jar. If the ball is white, no more balls are selected. If the ball is red, then another ball is selected. The ball is replaced in the jar after each selection. The process is repeated until a white ball is selected. Let R be the number of balls selected up to and including the first white ball.

(i) State the distribution of R and give the values of any parameters. [2]

(ii) State the value of $\mathrm{E}(R)$. [1]

(b) Suppose now that in the experiment described above, each selected ball is not replaced in the jar before the next selection is made. Let N be the number of balls selected up to and including the first white ball.

(i) Show that $\mathrm{P}(N = 2) = \frac{3}{10}$. [2]

(ii) The distribution of N is given in the table below.

n	1	2	3	4
$\mathrm{P}(N = n)$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{10}$

Find $\mathrm{E}(N)$ and $\mathrm{Var}(N)$. [5]

Geometric (cont 2)

6 Alfie and Betty play rounds of a game by each tossing two unbiased coins. A round results in a 'matching' if either they both obtain two heads, or they both obtain two tails or they both obtain a head and a tail.

Jan 05

(i) Show that the probability that the first round results in a matching is $\frac{3}{8}$. [3]

If the first round does not result in a matching, they continue to play rounds until a matching is obtained.

(ii) Find the probability that they play a total of 3 rounds or more. [4]

(iii) Find the expectation of the total number of rounds played, and give its meaning in the context of the question. [2]

7 Adill and Beth are playing a game. Adill throws a fair die three times. The number of sixes that he obtains is denoted by A. Beth throws a fair coin repeatedly. The number of throws up to and including the first throw on which the coin lands head upwards is denoted by B.

Jan 02

(i) State the distribution of A, giving the values of any parameters. [2]

(ii) State the distribution of B, giving the values of any parameters. [2]

(iii) Find $P(A = 2)$. [2]

(iv) Find $P(B > 2)$. [2]

(v) Find $P(A = B)$. [5]