Random Variables
Binomial Distribution

Spec

7 Items from a production line are examined for any defects. The probability that any item will be found to be defective is 0.15, independently of all other items.

(i) A batch of 16 items is inspected. Using tables of cumulative binomial probabilities, or otherwise, find the probability that

(a) at least 4 items in the batch are defective, [2]

(b) exactly 4 items in the batch are defective. [2]

(ii) Five batches, each containing 16 items, are taken.

(a) Find the probability that at most 2 of these 5 batches contain at least 4 defective items. [4]

(b) Find the expected number of batches that contain at least 4 defective items. [2]

1 The random variable X has a B(18, 0.4) distribution. Using the tables of cumulative binomial probabilities, and giving your answers correct to 3 significant figures, find

Jan 02

(i) $P(X \leqslant 7)$, [1]

(ii) $P(X > 6)$, [2]

(iii) $P(8 < X \leqslant 14)$. [3]

6 Every day I try to 'log on' to the internet. Over 100 days I found that I was successful at the first attempt on 88 days. I will try to log on to the internet each day for the next seven days. Let S be the number of days, out of the seven, on which I am successful at the first attempt.

Jun 03

(i) Suggest a model for the distribution of S, giving the values of any parameters. [2]

(ii) State two assumptions, in context, which are required to make this a good model. [2]

(iii) Calculate $P(S = 4)$. [3]

(iv) Calculate $P(S > E(S))$. [3]

1 A university graduate applied for 10 jobs after she gained her degree. The probability of her being offered any particular job is 0.2, independently of any other job. Let X be the number of jobs which she is offered.

Jan 03

(i) The distribution of X is B(n, p). State the values of the parameters n and p. [1]

(ii) State the value of E(X). [1]

(iii) Use the tables of cumulative binomial probabilities to find

(a) $P(X \leqslant 4)$,

(b) $P(2 \leqslant X < 6)$. [2]

Binomial (cont 1)

2 (i) The random variable X has a B(12, 0.6) distribution. Using the tables of cumulative binomial probabilities, find

NOV 03

(a) $P(X \leqslant 8)$, [1]

(b) $P(4 \leqslant X \leqslant 7)$. [3]

(ii) The random variable Y has a B(7, 0.43) distribution. Find $P(Y = 4)$. [3]

4 (a) The random variable W has a B(6, 0.7) distribution. Calculate the probability that the value of W is 3 or 6. [3]

NOV 02

(b) The random variable V has a B(9, 0.8) distribution. The mean of this distribution is denoted by μ and the standard deviation by σ.

(i) Find the value of μ. [2]

(ii) Use the table of cumulative binomial probabilities to find $P(V < \mu - \sigma)$. [3]

6 Sheena travels to work by car. From long observation, she has found that she can park in her favourite parking space on 2 days out of 5 on average. Let X be the number of days out of a 5-day working week on which she can park in her favourite parking space.

(i) State two assumptions which need to be made for a binomial model to be valid for the distribution of the random variable X. [2]

JUN 01

(ii) Assuming that B(n, p) is a valid model for the distribution of X,

(a) state the values of the parameters n and p, [2]

(b) show that $P(X > 3) = 0.0870$ correct to 3 significant figures. [2]

(iii) A 5-day working week in which Sheena can park in her favourite parking space on more than 3 days is a 'good' week. Find the probability that, out of 7 randomly chosen 5-day working weeks, fewer than 2 are good weeks. [4]

7 Wall tiles of a certain make are packed in boxes of 20. Production procedures lead to imperfections in 3% of these tiles, on average. A box of tiles is classified as 'unsatisfactory' if it contains more than one imperfect tile.

(i) What must be assumed for the number of imperfect tiles in a box to have a binomial distribution? [2]

JUN 05

(ii) Calculate the probability that a randomly chosen box is unsatisfactory. [4]

Each day randomly chosen boxes are checked. The number of boxes checked, up to and including the first unsatisfactory box, is denoted by U.

(iii) Calculate $E(U)$. [2]

(iv) If $U \geqslant 2 + E(U)$ then production is 'under control'. Calculate the probability that production is under control. [4]

Binomial (cont 2)

3 A company employs a large number of people in its city office and records show that 35% of the employees live outside the city limits. The Finance Department employs 18 men and 23 women. The number of these men who live outside the city limits is denoted by X and the number of these women who live outside the city limits is denoted by Y.

Jan 05

(i) Assuming a binomial model, find $P(6 \leqslant X \leqslant 10)$. [3]

(ii) Assuming a binomial model, find $P(Y = 10)$. [3]

(iii) Give a reason why binomial models might not be suitable. [1]

8 On average 8% of the student population is vegetarian.

(i) A party of 20 students goes on a field trip to a study centre in Scotland. The catering manager of the centre decides to prepare 2 vegetarian meals.

Nov 04

(a) Use a binomial distribution to find the probability that there are at most 2 vegetarians in the party of 20 students. [4]

(b) State two assumptions needed for the use of a binomial distribution in part (a). [2]

(ii) A group of n students goes on an exchange visit to Germany. The probability that this group contains at least one vegetarian is greater than 95%. Using trial and improvement, or otherwise, find the smallest possible value of n. [3]

5 Andy plays a lottery game once a week for 10 weeks. He knows that he has a probability of $\frac{1}{57}$ of winning each time he plays. Let X be the number of weeks out of 10 in which Andy wins.

(i) State the distribution of X, giving the values of any parameters, and state one assumption required to use this distribution as a suitable model. [3]

Jun 04

(ii) Calculate

(a) $P(X = 2)$, [3]

(b) $P(X > 2)$. [3]

(iii) Write down the value of $E(X)$. [1]

7 (a) The random variable W has a B(6, 0.4) distribution. Calculate the probability that the value of W is an odd number. [4]

(b) The random variable X has a B(5, p) distribution, where $p \neq 0$. Given that $E(X) = 3\text{Var}(X)$, find $P(X = 0)$. [3]

Jan 01

(c) The random variable Y has a B(12, p) distribution, where $p \neq 0$. Given that

$$P(Y = 11) = P(Y = 12),$$

find the value of p. [3]