4726 Further Pure Mathematics 2

1	(i)	Get $f'(x) = \pm \sin x/(1+\cos x)$	M1	Reasonable attempt at chain at any stage
		Get $f''(x)$ using quotient/product rule	M1	Reasonable attempt at quotient/product
		Get $f(0) = \ln 2$, $f'(0) = 0$, $f''(0) = -½$	B1	Any one correct from correct working
			A1	All three correct from correct working
	(ii)	Attempt to use Maclaurin correctly	M1	Using their values in $af(0)+bf'(0)x+cf''(0)x^2$; may be implied
		Get $\ln 2 - ¼x^2$	A1√	From their values; must be quadratic
2	(i)	Clearly verify in $y = \cos^{-1}x$	B1	i.e. $x=½\sqrt{3}$, $y=\cos^{-1}(½\sqrt{3})= {}^1/_6\pi$, or similar
		Clearly verify in $y = ½\sin^{-1}x$	B1	Or solve $\cos y = \sin 2y$
			SR	Allow one B1 if not sufficiently clear detail
	(ii)	Write down at least one correct diff'al	M1	Or reasonable attempt to derive; allow ±
		Get gradient of –2	A1	cao
		Get gradient of 1	A1	cao
3	(i)	Get y- values of 3 and $\sqrt{28}$	B1	
		Show/explain areas of two rectangles equal y- value x 1, and relate to A	B1	Diagram may be used
	(ii)	Show $A>0.2(\sqrt{(1+2^3)} + \sqrt{(1+2.2^3)} + \ldots$ ..$\sqrt{(1+2.83)}$)	M1	Clear areas attempted below curve (5 values)
		= 3.87(28)	A1	To min. of 3 s.f.
		Show $A<0.2(\sqrt{(1+2.2^3)} + \sqrt{(1+2.4^3)} + \ldots$ …+ $\sqrt{(1+3^3)}$)	M1	Clear areas attempted above curve (5 values)
		= 4.33(11) <4.34	A1	To min. of 3 s.f.
4	(i)	Correct formula with correct r	M1	May be implied
		Expand r^2 as A + B$\sec\theta$ + C$\sec^2\theta$	M1	Allow B = 0
		Get C $\tan\theta$	B1	
		Use correct limits in their answer	M1	Must be 3 terms
		Limits to ${}^1/_{12}\pi + 2\ln(\sqrt{3}) + {}^{2\sqrt{3}}/_3$	A1	AEEF; simplified
	(ii)	Use $x=r\cos\theta$ and $r^2 = x^2+y^2$	B1	Or derive polar form from given equation
		Eliminate r and θ	M1	Use their definitions
		Get $(x-2)\sqrt{(x^2+y^2)} = x$	A1	A.G.

5	(i)	Attempt use of product rule	M1	
		Clearly get $x=1$	A1	Allow substitution of x=1
	(ii)	Explain use of tangent for next approx.	B1	Not use of G.C. to show divergence
		Tangents at successive approx. give $x>1$	B1	Relate to crossing x-axis; allow diagram
	(iii)	Attempt correct use of N-R with their derivative	M1	
		Get $x_2 = -1$	A1√	
		Get –0.6839, -0.5775, (-0.5672…)	A1	To 3 d.p. minimum
		Continue until correct to 3 d.p.	M1	May be implied
		Get –0.567	A1	cao
6	(i)	Attempt division/equate coeff.	M1	To lead to some $ax+b$ (allow b=0 here)
		Get $a = 2$, $b = -9$	A1	
		Derive/quote $x = 1$	B1	Must be equations
	(ii)	Write as quadratic in x	M1	$(2x^2-x(11+y)+(y-6)=0)$
		Use $b^2 \geq 4ac$ (for real x)	M1	Allow <, >
		Get $y^2 + 14y + 169 \geq 0$	A1	
		Attempt to justify positive/negative	M1	Complete the square/sketch
		Get $(y+7)^2 + 120 \geq 0$ – true for all y	A1	
			SC	Attempt diff; quot./prod. rule M1 Attempt to solve $dy/dx = 0$ M1 Show $2x^2 - 4x + 17 = 0$ has no real roots e.g. $b^2 - 4ac < 0$ A1 Attempt to use no t.p. M1 Justify all y e.g. consider asymptotes and approaches A1
7	(i)	Get $x(1+x^2)^{-n} - \int x.(-n(1+x^2)^{-n-1}.2x)\,dx$	M1	Reasonable attempt at parts
		Accurate use of parts	A1	
		Clearly get A.G.	B1	Include use of limits seen
	(ii)	Express x^2 as $(1+x^2) - 1$		
		Get $\frac{x^2}{(1+x^2)^{n+1}} = \frac{1}{(1+x^2)^n} - \frac{1}{(1+x^2)^{n+1}}$	B1	Justified
		Show $I_n = 2^{-n} + 2n(I_n - I_{n+1})$	M1	Clear attempt to use their first line above
		Tidy to A.G.	A1	
	(iii)	See $2I_2 = 2^{-1} + I_1$	B1	
		Work out $I_1 = \frac{1}{4}\pi$	M1	Quote/derive $\tan^{-1}x$
		Get $I_2 = \frac{1}{4} + \frac{1}{8}\pi$	A1	

8	**(i)**	Use correct exponential for sinh x	B1	
		Attempt to expand cube of this	M1	Must be 4 terms
		Correct cubic	A1	
		Clearly replace in terms of sinh	B1	(Allow RHS→ LHS or RHS = LHS separately)
	(ii)	Replace and factorise	M1	Or state sinh $x \neq 0$
		Attempt to solve for $\sinh^2 x$	M1	(= ¼(k-3)) or for k and use $\sinh^2 x>0$
		Get $k>3$	A1	Not ≥
	(iii)	Get $x = \sinh^{-1} c$	M1	(c= ±½); allow sinh $x = c$
		Replace in ln equivalent	A1√	As ln(½+√ $^5/_4$); their x
		Repeat for negative root	A1√	May be given as neg. of first answer (no need for x=0 implied)
			SR	Use of exponential definitions Express as cubic in $e^{2x} = u$ M1 Factorise to $(u-1)(u^2-3u+1)=0$ A1 Solve for x =0, ½ln($^3/_2$ ±$^{\sqrt{5}}/_2$) A1
9	**(i)**	Get sinh y $^{dy}/_{dx}$ =1	M1	Or equivalent; allow ± Allow use of ln equivalent with Chain Rule
		Replace sinh $y = \sqrt{(\cosh^2 y - 1)}$	A1	
		Justify positive grad. to A.G.	B1	e.g. sketch
	(ii)	Get $k \cosh^{-1} 2x$	M1	No need for c
		Get k=½	A1	
	(iii)	Sub. $x = k \cosh u$	M1	
		Replace all x to $\int k_1 \sinh^2 u \, du$	A1	
		Replace as $\int k_2(\cosh 2u - 1) \, du$	M1	Or exponential equivalent
		Integrate correctly	A1√	No need for c
		Attempt to replace u with x equivalent	M1	In their answer
		Tidy to reasonable form	A1	cao (½$x\sqrt{(4x^2 - 1)}$ - ¼ $\cosh^{-1} 2x$ (+c))