<u>OCR – Core Mathematics 3 January 2013</u>

1. (i)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x+1)\times 3 - 3x \times 2}{(2x+1)^2}$$

$$= \frac{3}{(2x+1)^2}$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=2} = \frac{3}{(2\times 2+1)^2} = \frac{3}{5^2} = \frac{3}{25}$$

(ii)

$$y = (4x^2+9)^{\frac{1}{2}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}(4x^2+9)^{-\frac{1}{2}} \times 8x$$

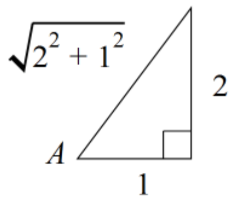
$$= 4x(4x^2+9)^{-\frac{1}{2}}$$

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=2} = 4\times 2\times(4\times 2^2+9)^{-\frac{1}{2}}$$

$$= 8\times 25^{-\frac{1}{2}} = 8\times\frac{1}{\sqrt{25}} = \frac{8}{5}$$

2. (i)

$$\tan A = 2 = \frac{2}{1}$$



$$\sin A = \frac{2}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}}$$

$$\operatorname{cosec} A = \frac{\sqrt{5}}{2}$$

(ii)

$$\tan(A+B) = 3$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 3$$

$$\frac{2+\tan B}{1-2\tan B} = 3$$

$$2 + \tan B = 3(1-2\tan B)$$

$$2+\tan B = 3 - 6\tan B$$

$$7\tan B = 1$$

$$\tan B = \frac{1}{7}$$

3. (a) $|t| = 3 \Rightarrow t = -3$ or $t = 3$

When $t = -3$, $|2t - 1| = |2 \times -3 - 1| = |-7| = 7$

When $t = 3$, $|2t - 1| = |2 \times 3 - 1| = |5| = 5$

(b)

$$\left|x - \sqrt{2}\right| > \left|x + 3\sqrt{2}\right|$$

$$(x - \sqrt{2})^2 > (x + 3\sqrt{2})^2$$

$$x^2 - 2\sqrt{2}x + 2 > x^2 + 6\sqrt{2}x + 3^2 \times (\sqrt{2})^2$$

$$-2\sqrt{2}x + 2 > 6\sqrt{2}x + 18$$

$$8\sqrt{2}x < -16$$

$$\sqrt{2}x < -2$$

$$x < -\frac{2}{\sqrt{2}}$$

$$x < -\sqrt{2}$$

4. (i) $e^{0.021t} = 2$

$$0.021t = \ln 2$$

$$t = \frac{\ln 2}{0.021} = 33.0 \text{ hours (3 sf)}$$

Doubling every 33.0 hours means it has to double twice more to reach 8 times so it will take $3 \times 33.0 = 99.0$ hours (3 sf)

(ii)

When $m = 400$

$$250e^{0.021t} = 400$$

$$e^{0.021t} = \frac{400}{250} = \frac{8}{5}$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 250 \times 0.021e^{0.021t} = 5.25e^{0.021t}$$

$$\left.\frac{\mathrm{d}m}{\mathrm{d}t}\right|_{m=400} = 5.25 \times \frac{8}{5} = 8.4 \text{ grams/hour}$$

5. (i)

$$\text{Area} = \int_2^9 \frac{6}{\sqrt{3x+1}}\,\mathrm{d}x$$

$$= \int_2^9 6(3x+1)^{-\frac{1}{2}}\,\mathrm{d}x$$

$$= \left[\frac{6(3x+1)^{\frac{1}{2}}}{\frac{1}{2}\times 3}\right]_2^9$$

$$= \left[4\sqrt{3x+1}\right]_2^9$$

$$= 4\sqrt{3\times 9+1} - 4\sqrt{3\times 2+1}$$

$$= 4\sqrt{28} - 4\sqrt{7}$$

$$= 4\times 2\sqrt{7} - 4\sqrt{7}$$

$$= 8\sqrt{7} - 4\sqrt{7}$$

$$= 4\sqrt{7} \text{ square units}$$

(ii)

$$\text{Volume} = \int_2^9 \pi\left(\frac{6}{\sqrt{3x+1}}\right)^2 \mathrm{d}x$$

$$= \int_2^9 \frac{36\pi}{3x+1}\,\mathrm{d}x$$

$$= \left[\frac{36\pi}{3}\ln|3x+1|\right]_2^9$$

$$= 12\pi\ln|3\times 9+1| - 12\pi\ln|3\times 2+1|$$

$$= 12\pi\ln 28 - 12\pi\ln 7$$

$$= 12\pi\ln\left(\frac{28}{7}\right)$$

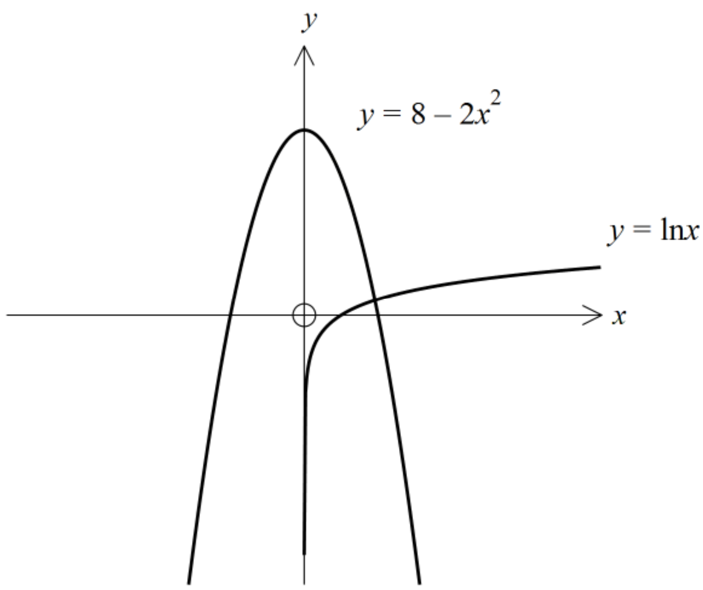
$$= 12\pi\ln 4$$

$$= 12\pi\ln 2^2$$

$$= 12\pi\times 2\ln 2$$

$$= 24\pi\ln 2$$

6. (i)



Since graphs only meet once there is only one root of $\ln x = 8 - 2x^2$

(ii)

$y = \ln x$ meets the x-axis when $x = 1$

$y = 8 - 2x^2$ meets the x-axis when

$$8 - 2x^2 = 0$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \pm 2$$

Hence root between $x = 1$ and $x = 2$ since the curves intersect between these two value

(iii)

$x_0 = 1.5$

$x_1 = \sqrt{4 - 0.5\ln 1.5} = 1.948657858$

$x_2 = \sqrt{4 - 0.5\ln 1.948657858} = 1.914792305$

$x_3 = 1.91707992$

$x_4 = 1.916924209$

$x_5 = 1.916934802$

$x = 1.917$ (3 dp)

(iv) Currently meet at $(1.917, \ln 1.917)$

Translation makes curves meet at $(3.917, \ln 1.917)$

Stretch makes curves meet at $(3.917, 4\ln 1.917)$

New curves meet at $(3.92, 2.60)$ (2 dp)

7. (i) $\dfrac{dx}{dy} = (y+4)\dfrac{2}{2y+3} + 1\ln(2y+3)$

$$= \frac{2(y+4)}{2y+3} + \ln(2y+3)$$

(ii)

$$\text{At } A,\ y = 0$$

$$\frac{dx}{dy} = \frac{2(0+4)}{2\times 0+3} + \ln(2\times 0+3)$$

$$= \frac{8}{3} + \ln 3$$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{8}{3} + \ln 3\right)} = 0.27 \text{ (2 dp)}$$

$$\text{At } B,\ x = 0$$

$$(y+4)\ln(2y+3) = 0$$

$$y+4 = 0 \text{ or } \ln(2y+3) = 0$$

$$y = -4$$

Impossible since $\ln(2\times -4+3) = \ln(-5)$ which does not exist

$$2y+3 = 1$$

$$y = -1$$

$$\frac{dx}{dy} = \frac{2(-1+4)}{2\times -1+3} + \ln(2\times -1+3)$$

$$= 6$$

$$\frac{dy}{dx} = \frac{1}{6}$$

8. (i) $$f(x) = (x+2a)^2 + a^2 - (2a)^2$$

$$= (x+2a)^2 - 3a^2$$

Range is $f(x) \geq -3a^2$

(ii)

$$fg(x) = (4x-2a)^2 + 4a(4x-2a) + a^2$$

$$= 16x^2 - 16ax + 4a^2 + 16ax - 8a^2 + a^2$$

$$= 16x^2 - 3a^2$$

$$fg(3) = 69$$

$$16 \times 3^2 - 3a^2 = 69$$

$$144 - 3a^2 = 69$$

$$3a^2 = 75$$

$$a^2 = 25$$

$a = 5$ since $a > 0$

$$g(x) = 4x - 10$$

$$g^{-1}(x) = \frac{x+10}{4}$$

$$g^{-1}(x) = x$$

$$\frac{x+10}{4} = x$$

$$x + 10 = 4x$$

$$3x = 10$$

$$x = \frac{10}{3}$$

9. (i)

$$\text{LHS} = (\cos\theta\cos 45° - \sin\theta\sin 45°)^2 - \tfrac{1}{2}(1 - 2\sin^2\theta - \sin 2\theta)$$

$$= \cos^2\theta\cos^2 45° - 2\sin 45°\cos 45°\sin\theta\cos\theta + \sin^2\theta\sin^2 45° - \tfrac{1}{2} + \sin^2\theta + \tfrac{1}{2}\sin 2\theta$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2\cos^2\theta - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\sin 2\theta + \left(\frac{1}{\sqrt{2}}\right)^2\sin^2\theta - \tfrac{1}{2} + \sin^2\theta + \tfrac{1}{2}\sin 2\theta$$

$$= \tfrac{1}{2}(\cos^2\theta + \sin^2\theta) \cancel{-\tfrac{1}{2}\sin 2\theta} - \tfrac{1}{2} + \sin^2\theta \cancel{+\tfrac{1}{2}\sin 2\theta}$$

$$= \tfrac{1}{2}\times 1 - \tfrac{1}{2} + \sin^2\theta$$

$$= \sin^2\theta = \text{RHS}$$

(ii)

$$6\sin^2\tfrac{1}{2}\theta = 2$$

$$\sin^2\tfrac{1}{2}\theta = \frac{1}{3}$$

$$\sin\tfrac{1}{2}\theta = \pm\frac{1}{\sqrt{3}}$$

$$\tfrac{1}{2}\theta = 35.3° \text{ or } -35.3°$$

$$\theta = 70.5° \text{ or } -70.5° \text{ (3 sf)}$$

(iii)

$$6\sin^2\tfrac{1}{3}\theta = k$$

$$\sin^2\tfrac{1}{3}\theta = \frac{k}{6}$$

$$0 < \tfrac{1}{3}\theta < 30°$$

$$0 < \sin^2\tfrac{1}{3}\theta < \sin^2 30°$$

$$0 < \frac{k}{6} < \left(\frac{1}{2}\right)^2$$

$$0 < \frac{k}{6} < \frac{1}{4}$$

$$0 < k < \frac{3}{2}$$